Dynamic Multistate Population Models

Peter W.F. Smith

University of Southampton

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  - with two states
  - with age
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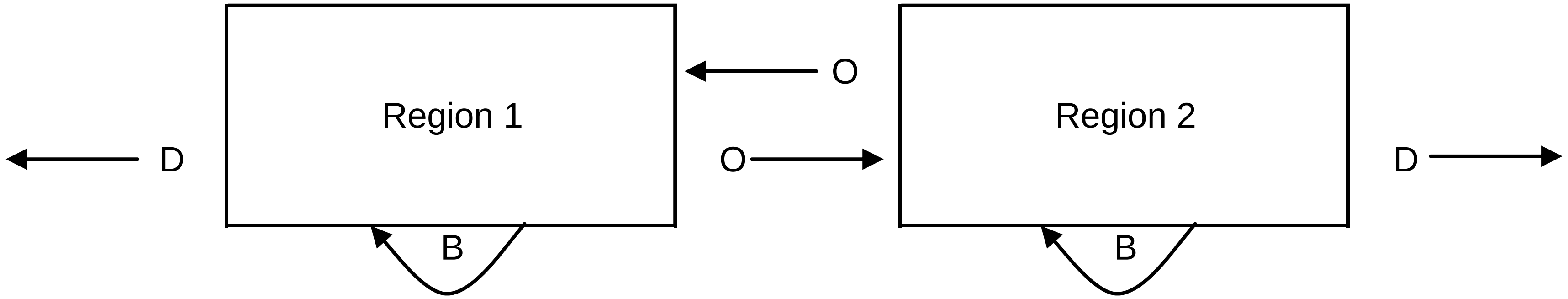
Introduction

- Dynamic multistate population models
  - provide a general and flexible platform for modelling and analysing population change over time
  - allow the combination of all the main components of population change by age and sex with various transitions that population groups may experience throughout their life course

Introduction

- These transitions may include those between states of residences, employment, marriage or health
- The multistate approach
  – avoids potential inconsistencies arising from inappropriately defined rates
    - e.g., the in-migration rate or the net migration rate
  – allows one to follow individuals or groups across several changes in their states

Matrix model – with two states



$$P_1^1 = P_1^0 + B_1 - D_1 - O_1 + O_2$$
$$= P_1^0(1 + b_1 - d_1 - o_1) + P_2^0 o_2$$

$$P_2^1 = P_2^0 + B_2 - D_2 - O_2 + O_1$$
$$= P_2^0(1 + b_2 - d_2 - o_2) + P_1^0 o_1$$

In matrix form

$$\boldsymbol{P}^1 = \begin{bmatrix} P_1^1 \\ P_2^1 \end{bmatrix} = \begin{bmatrix} 1 + b_1 - d_1 - o_1 & o_2 \\ o_1 & 1 + b_2 - d_2 - o_2 \end{bmatrix} \begin{bmatrix} P_1^0 \\ P_2^0 \end{bmatrix} = G\boldsymbol{P}^0$$

Matrix model – with two states

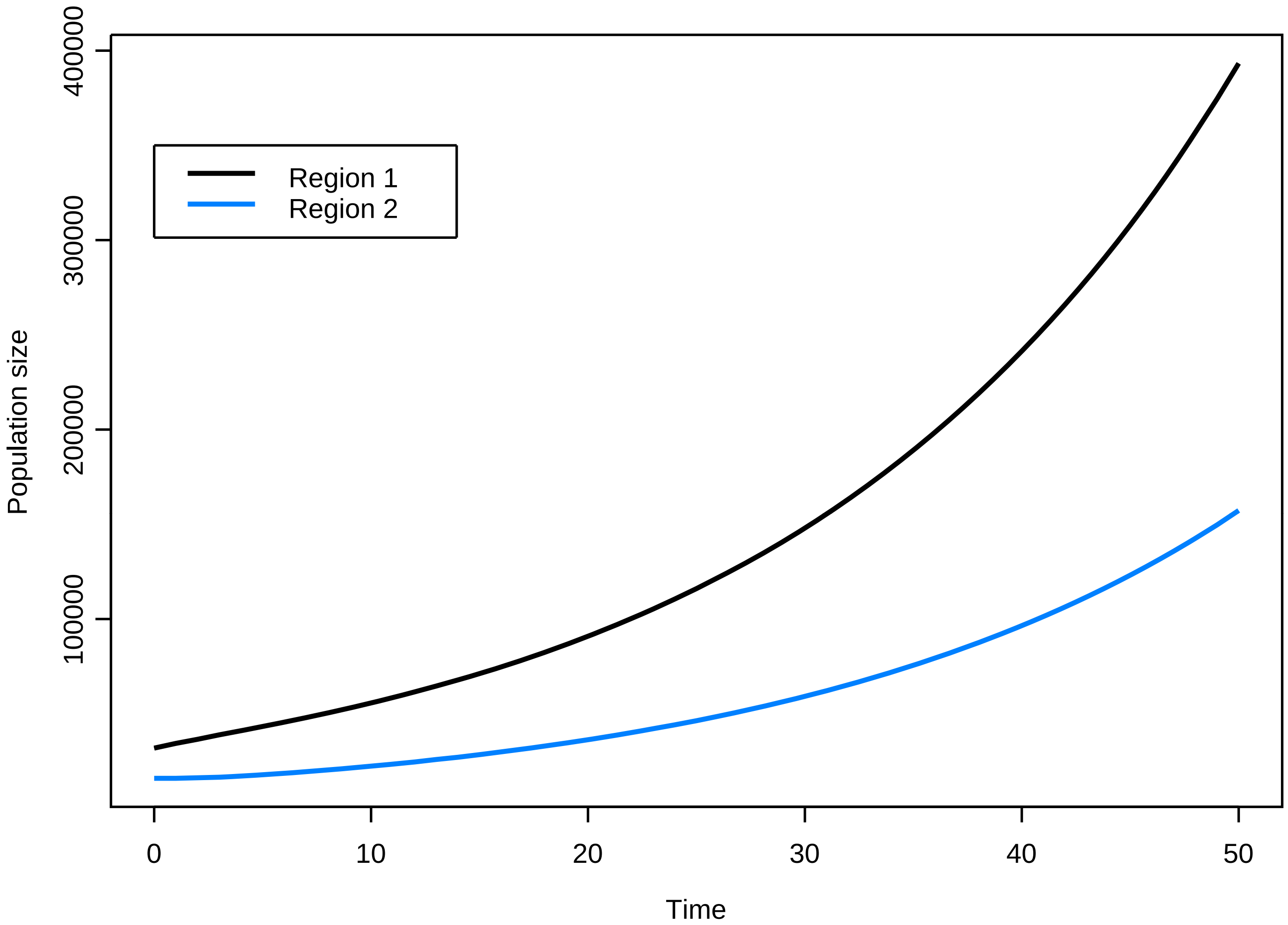
Numerical example

$$\boldsymbol{P}^1 = \begin{bmatrix} 1+0.06-0.01-0.10 & 0.25 \\ 0.10 & 1+0.06-0.01-0.25 \end{bmatrix} \begin{bmatrix} 32\,000 \\ 16\,000 \end{bmatrix} = \begin{bmatrix} 34\,400 \\ 16\,000 \end{bmatrix}$$

$$\boldsymbol{P}^2 = \begin{bmatrix} 0.95 & 0.25 \\ 0.10 & 0.80 \end{bmatrix} \begin{bmatrix} 34\,400 \\ 16\,000 \end{bmatrix} = \begin{bmatrix} 0.95 & 0.25 \\ 0.10 & 0.80 \end{bmatrix}^2 \begin{bmatrix} 32\,000 \\ 16\,000 \end{bmatrix} = \begin{bmatrix} 36\,680 \\ 16\,240 \end{bmatrix}$$

$$\boldsymbol{P}^n = \begin{bmatrix} 0.95 & 0.25 \\ 0.10 & 0.80 \end{bmatrix}^n \begin{bmatrix} 32\,000 \\ 16\,000 \end{bmatrix} = G^n \boldsymbol{P}^0$$

Matrix model – with two states



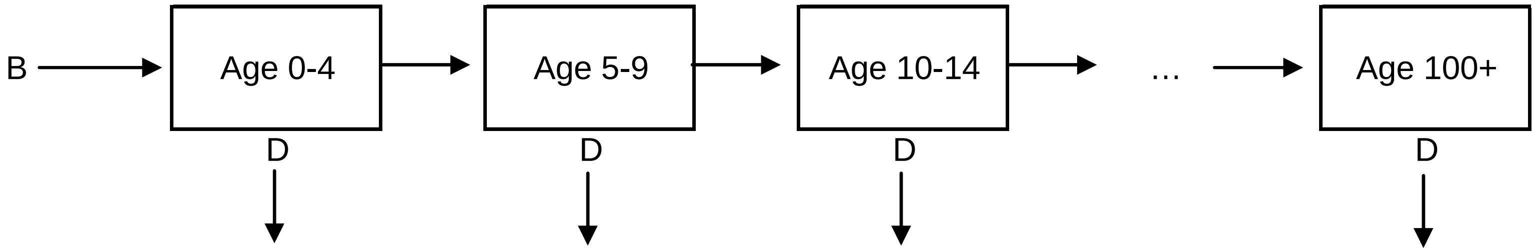
Matrix models

- Facilitates, for the stable population, calculation of
  - the growth rate
  - the population distribution across the states

  using matrix algebra
  - first eigenvalue and eigenvector of *G*
  - for numerical example r = 5% and p_1/p_2 = 2.5

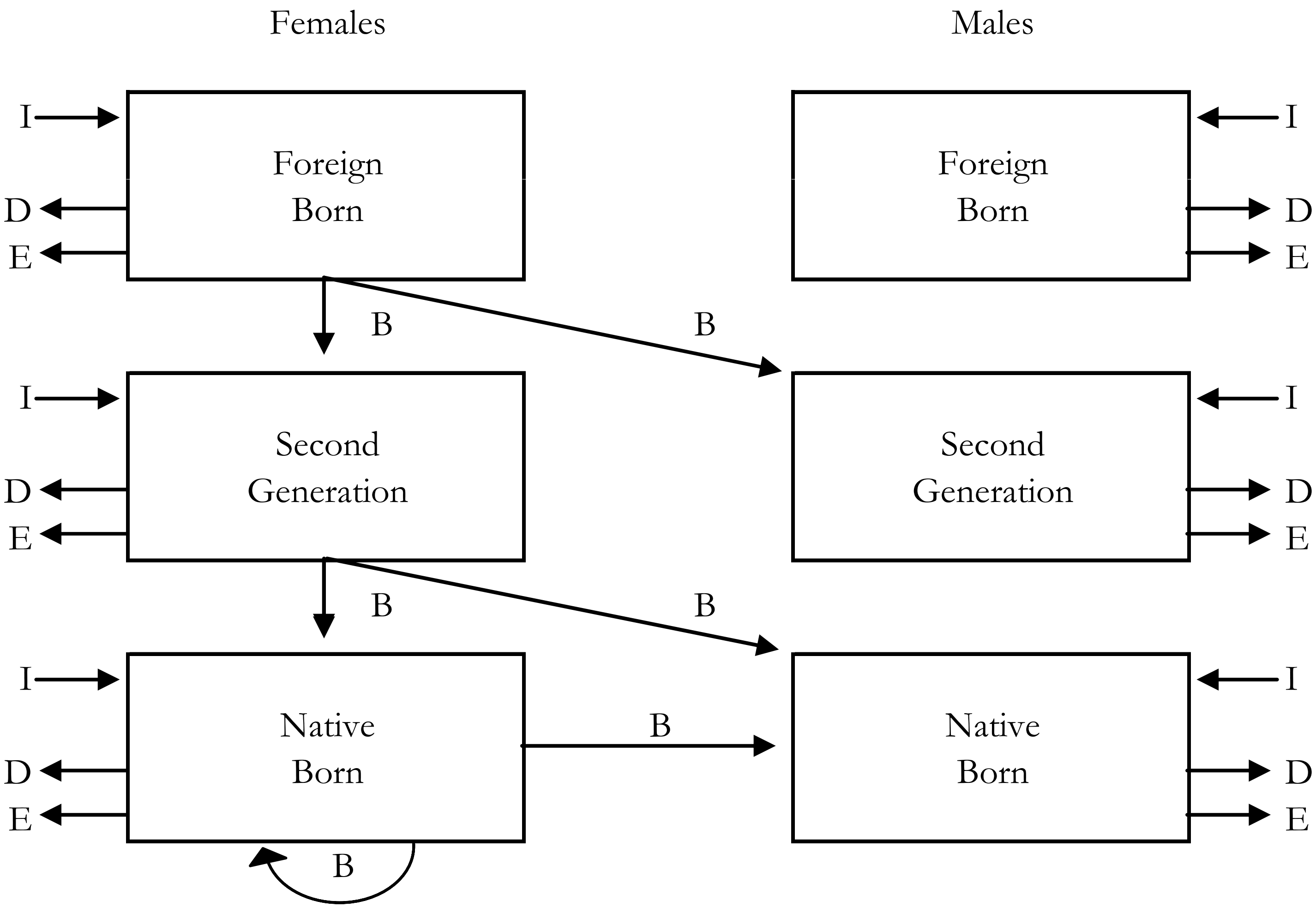
Matrix model – with age

Female



$$
\begin{bmatrix} P^1_{0-4} \\ P^1_{5-9} \\ P^1_{10-14} \\ P^1_{15-19} \\ P^1_{20-24} \\ P^1_{25-29} \\ \vdots \\ P^1_{100+} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & b_{15-19} & b_{20-24} & b_{25-29} & \cdots & 0 \\
1-d_{0-4} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 1-d_{5-9} & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1-d_{10-14} & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 1-d_{15-19} & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 1-d_{20-24} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1-d_{95-99} & 0
\end{bmatrix}
\begin{bmatrix} P^0_{0-4} \\ P^0_{5-9} \\ P^0_{10-14} \\ P^0_{15-19} \\ P^0_{20-24} \\ P^0_{25-29} \\ \vdots \\ P^0_{100+} \end{bmatrix}
$$

Matrix model – with three states



Matrix model – with three states

$$\boldsymbol{P}^{t+n} = \begin{bmatrix} P_{1f}^{t+n} \\ P_{2f}^{t+n} \\ P_{3f}^{t+n} \\ P_{1m}^{t+n} \\ P_{2m}^{t+n} \\ P_{3m}^{t+n} \end{bmatrix} = G^n \begin{bmatrix} P_{1f}^{t} \\ P_{2f}^{t} \\ P_{3f}^{t} \\ P_{1m}^{t} \\ P_{2m}^{t} \\ P_{3m}^{t} \end{bmatrix} + \begin{bmatrix} I_{1f} \\ I_{2f} \\ I_{3f} \\ I_{1m} \\ I_{2m} \\ I_{3m} \end{bmatrix} = G^n \boldsymbol{P}^t + \boldsymbol{I}$$

$$G = \begin{bmatrix} 1+b_{1f}-d_{1f}-e_{1f} & b_{2f} & 0 & 0 & 0 & 0 \\ 0 & 1-d_{2f}-e_{2f} & b_{3f} & 0 & 0 & 0 \\ 0 & 0 & 1-d_{3f}-e_{3f} & 0 & 0 & 0 \\ b_{1m} & b_{2m} & 0 & 1-d_{1m}-e_{1m} & 0 & 0 \\ 0 & 0 & b_{3m} & 0 & 1-d_{2m}-e_{2m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-d_{3m}-e_{3m} \end{bmatrix}$$

Modelling unknown population dynamics

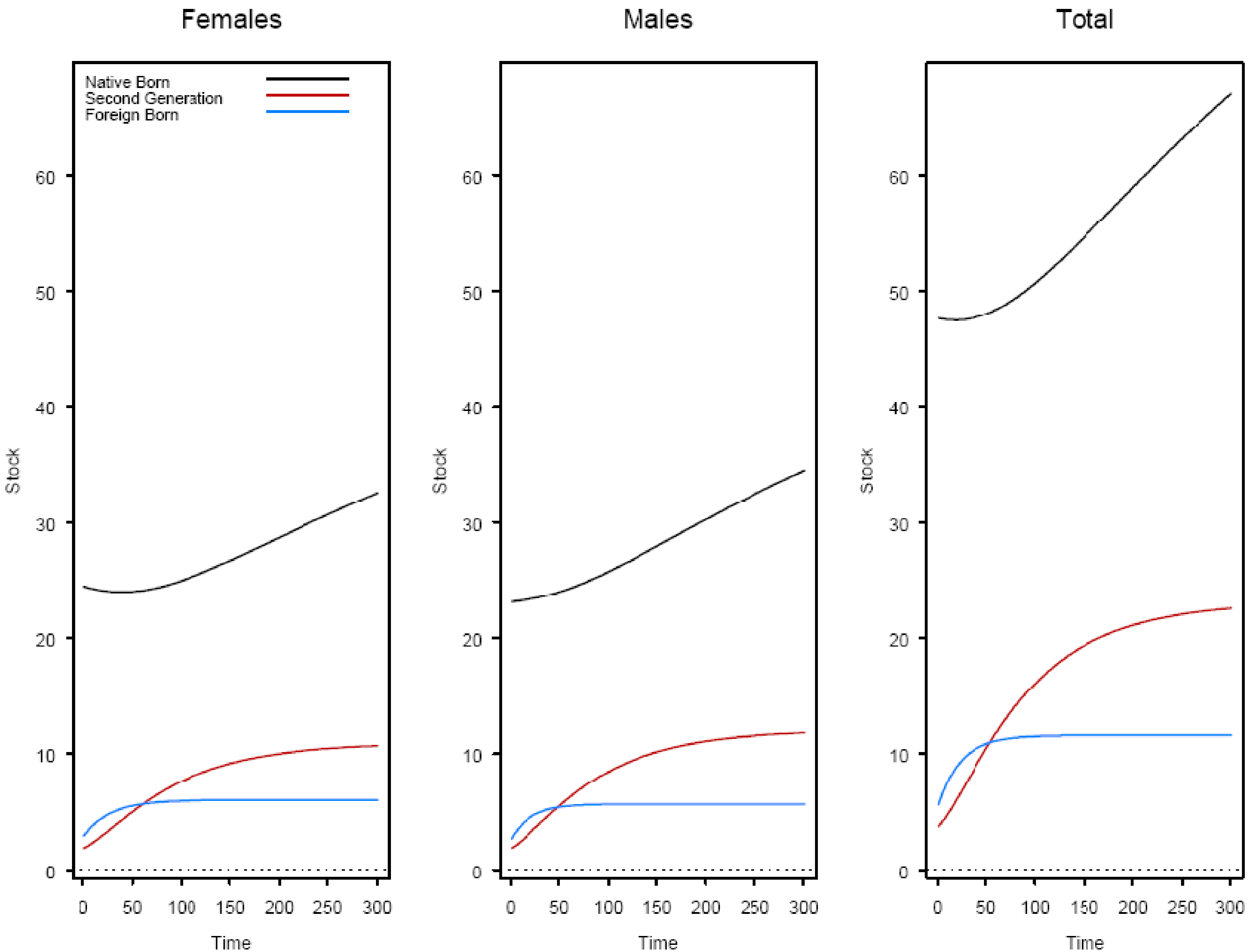
- There is very little demographic data on the second generation population
  - We only have some information on the stocks from the General Household Survey
  - We have nothing on births, deaths, immigration or emigration (only native-born and foreign-born)
    - First, assume that birth and death rates for second generation are the same as native-born

Modelling unknown population dynamics

- Disaggregate immigration into native-born and second generation with $\alpha = 0.93$

$$\begin{bmatrix} I_{1f} \\ I_{2f} \\ I_{3f} \\ I_{1m} \\ I_{2m} \\ I_{3m} \end{bmatrix} = \begin{bmatrix} 79\,229\alpha \\ 79\,229(1-\alpha) \\ 86\,423 \\ 107\,771\alpha \\ 107\,771(1-\alpha) \\ 106\,577 \end{bmatrix}$$

$$\boldsymbol{I} = \begin{bmatrix} 73\,683 \\ 5\,546 \\ 86\,423 \\ 100\,227 \\ 7\,544 \\ 106\,577 \end{bmatrix}$$



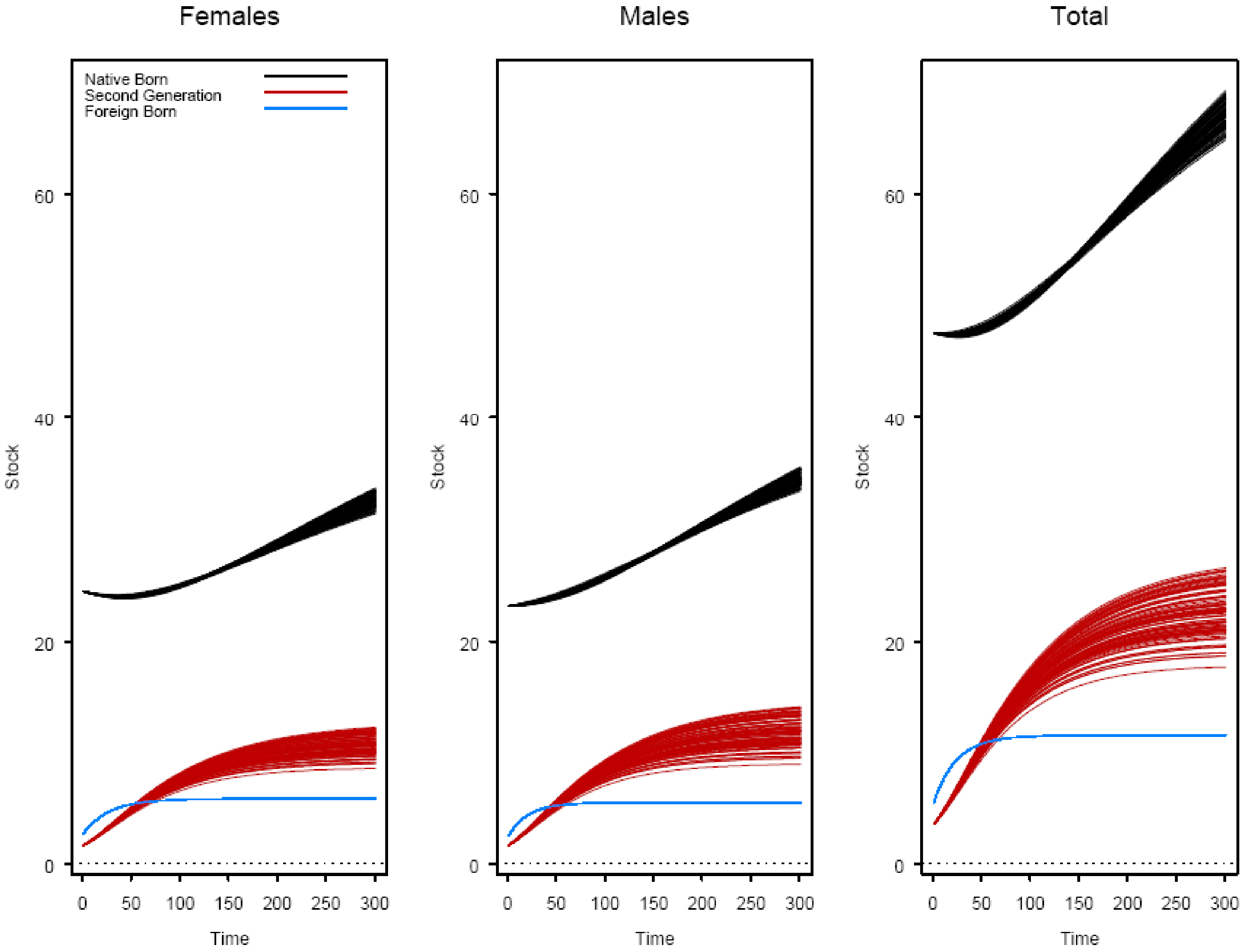
Females
Males
Total
Native Born
Second Generation
Foreign Born
Stock
Time
0
10
20
30
40
50
60
0
50
100
150
200
250
300

Including uncertainty

- Is important for understanding the implications of the quality of the data and our assumptions
- There are many ways to include uncertainty
  - Data uncertainty: allow alpha (in the second generation projection model) to follow a beta distribution
  - Model uncertainty: Bayesian models allow the estimation of parameters based on historical time series and expert judgements

Projections by nativity with uncertainty added to migration

$$\alpha \sim Be(93,7)$$



Extensions

CPC
centre for population change

- Include age, sex and multiple states
- Allow parameters to vary over time
- Resulting models will have many parameters (*b*'s, *d*'s, *o*'s, *α*'s, etc.) whose values are not known exactly
- Need to build models for them
  - e.g., for age schedules for fertility, migration and mortality

Conclusions

- Interested in the modelling of multistate populations in the context of incomplete data and uncertainty
- We are heading in the direction of Bayesian population projections, which allows us to quantify data and model uncertainty and incorporate expert judgements