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Bayesian Forecasting and Issues of Uncertainty

Jakub Bijak

University of Southampton

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Background

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Ongoing change of perspective in population forecasting: [Keilman 2001, Alho & Spencer 2005]

- Deterministic point forecasts or projections: Almost certainly will **NOT** come true
- Variant projections (Base / High / Low) do not have any probabilities associated with the variants
- **Solution** – Probabilistic forecasts, but:
  - How to quantify the uncertainty?
  - How to deal with inadequate data?
  - How to use the output?

Preliminaries

Approaches to probabilistic forecasting:

Classical ('*Frequentist*')

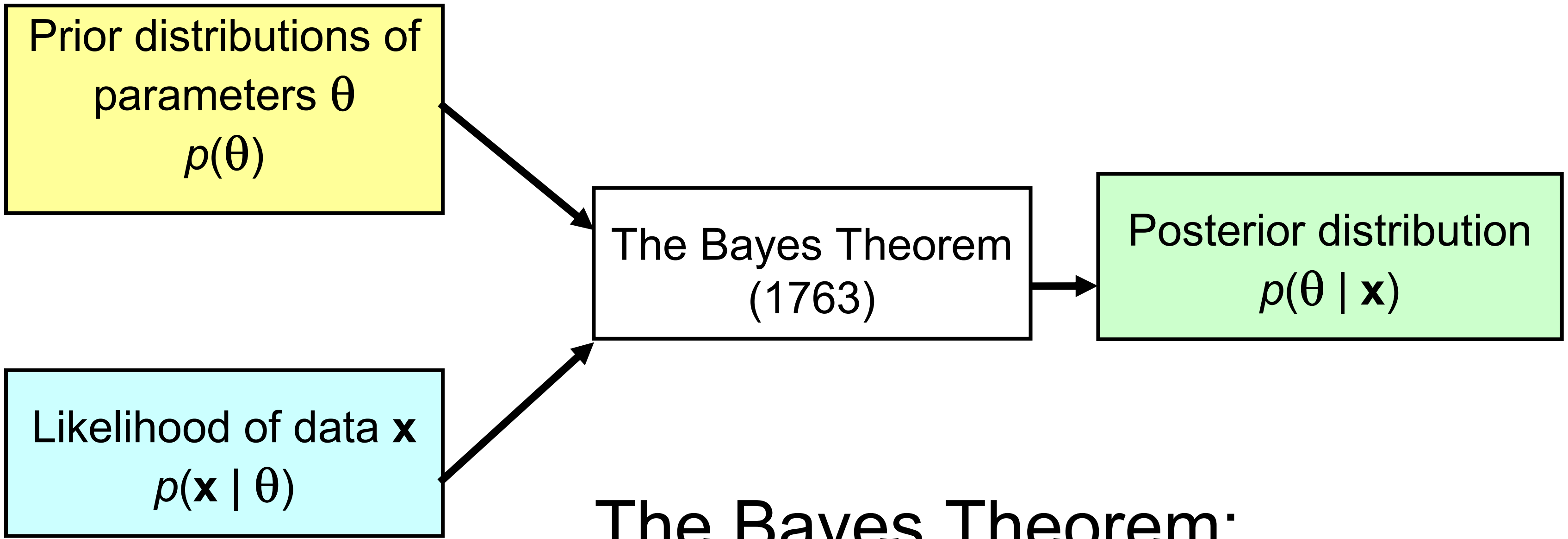
- Probability related to the frequency of events
- Model parameters are constant, yet unknown
- Inference based on the data at hand, assuming a given likelihood function

Bayesian

- Probability is a subjective measure of belief
- Model parameters are random and have probability distributions
- Inference based on the data at hand, assuming a given likelihood function and *prior distributions* for parameters

Preliminaries

- The Bayesian Approach



The Bayes Theorem:

$$p(\theta|\mathbf{x}) = p(\theta)\, p(\mathbf{x}|\theta) \,/\, p(\mathbf{x})$$

Preliminaries

- Prior information
  - Knowledge on the parameters independent from data
  - Can be based on the insights of the researcher, other experts in the field, or can be *non-informative*
- Essence of Bayesian inference
  - Update of the prior beliefs in the light of obtained evidence (data) to obtain the *posterior* information
  - Coherent inference within a joint probabilistic model
  - Natural extension to predictions

Bayesian estimation

- Bayesian estimates of parameters are based on the posterior distributions
- Three main approaches:
  - Orthodox Bayesianism
    - Posterior distribution is enough to describe the uncertainty of model parameters
  - Empirical Bayesianism
    - Prior distributions estimated from the empirical data
    - Criticised for incoherence
  - Decision-theoretic Bayesianism (*more later…*)

Bayesian estimation

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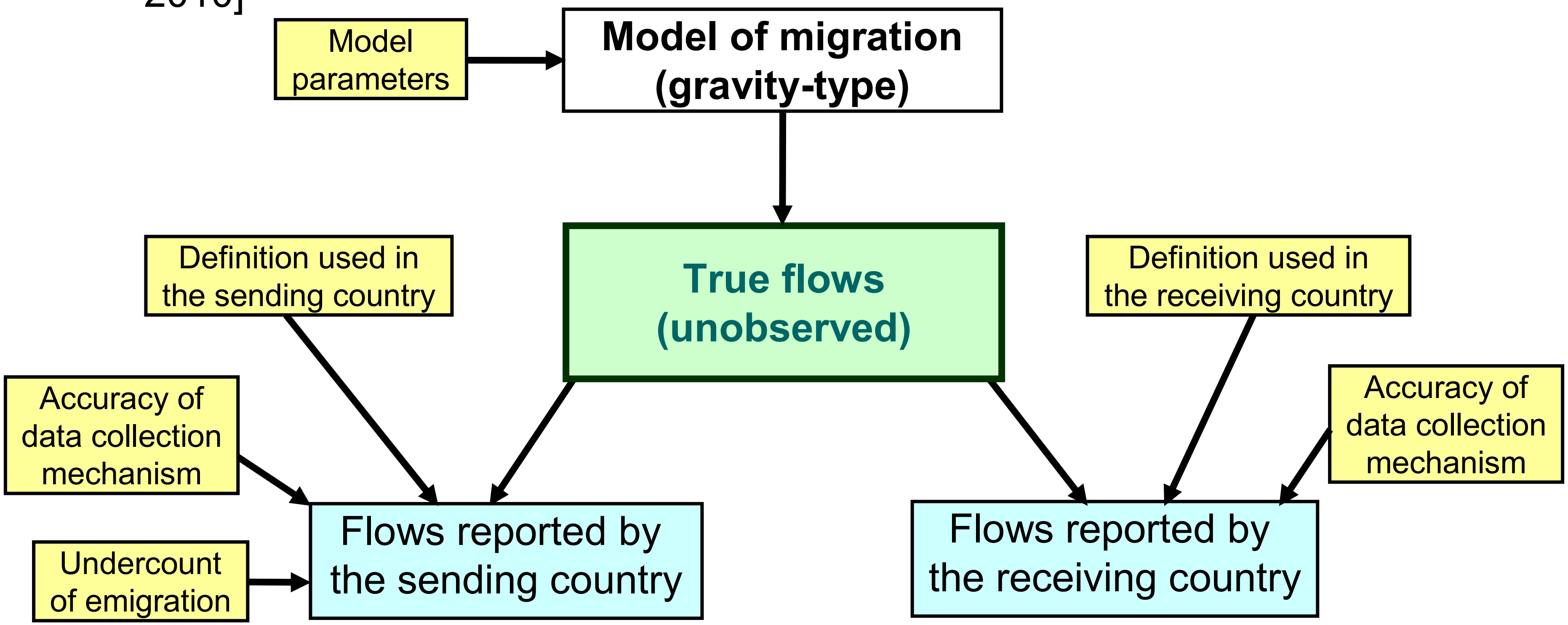
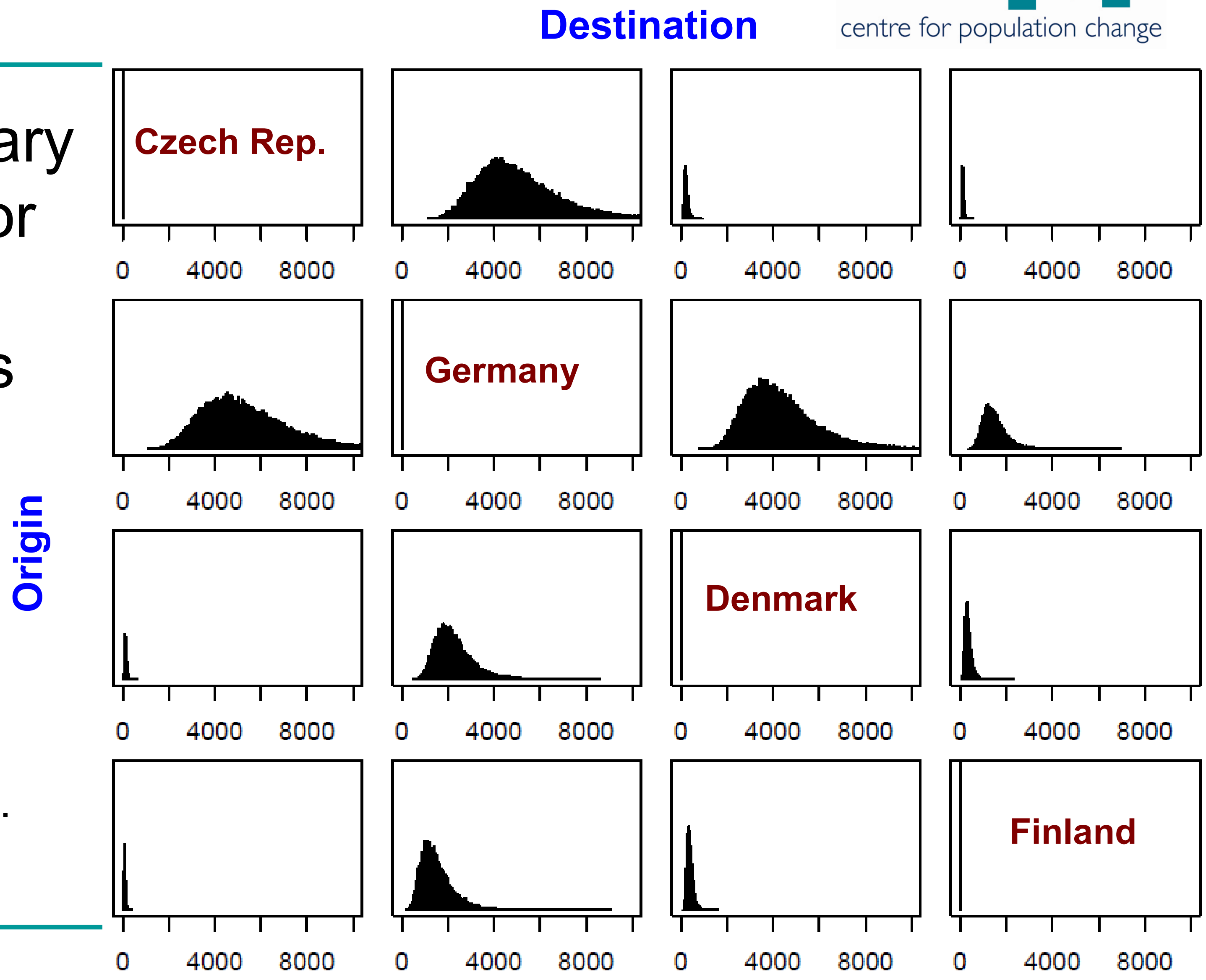
- **Example:** Model for estimating international migration flows given deficient data [Raymer *et al.* 2010]



Bayesian estimation

- Preliminary results for four EU countries

[Raymer et al. 2010]



Bayesian forecasting

- **Main idea:** the *predictive distribution* of the future ($\mathbf{x}^P$) given the past data ($\mathbf{x}$) follows directly from the joint probabilistic model
- Predictive distribution $p(\mathbf{x}^P|\mathbf{x})$ is derived from the sampling predictive density $p(\mathbf{x}^P|\mathbf{x},\theta)$, weighted by the posterior distribution $p(\theta|\mathbf{x})$ [e.g. Zellner 1971]
  - Formally:
    $$p(\mathbf{x}^P|\mathbf{x}) = \int_\Theta p(\mathbf{x}^P|\mathbf{x},\theta)\, p(\theta|\mathbf{x})\, d\theta$$

Bayesian forecasting

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- Additional possibility offered by the Bayesian approach: model selection and averaging
  - Takes into account the uncertainty of model specification, within a certain model class ***M***
  - Extended application of the Bayes theorem
    - Requires information *a priori* about the probabilities of particular models, $p(m)$
    - Allows to select the model with highest posterior probability $p(m|\mathbf{x})$, or average the forecasts over ***M***

Bayesian forecasting: example

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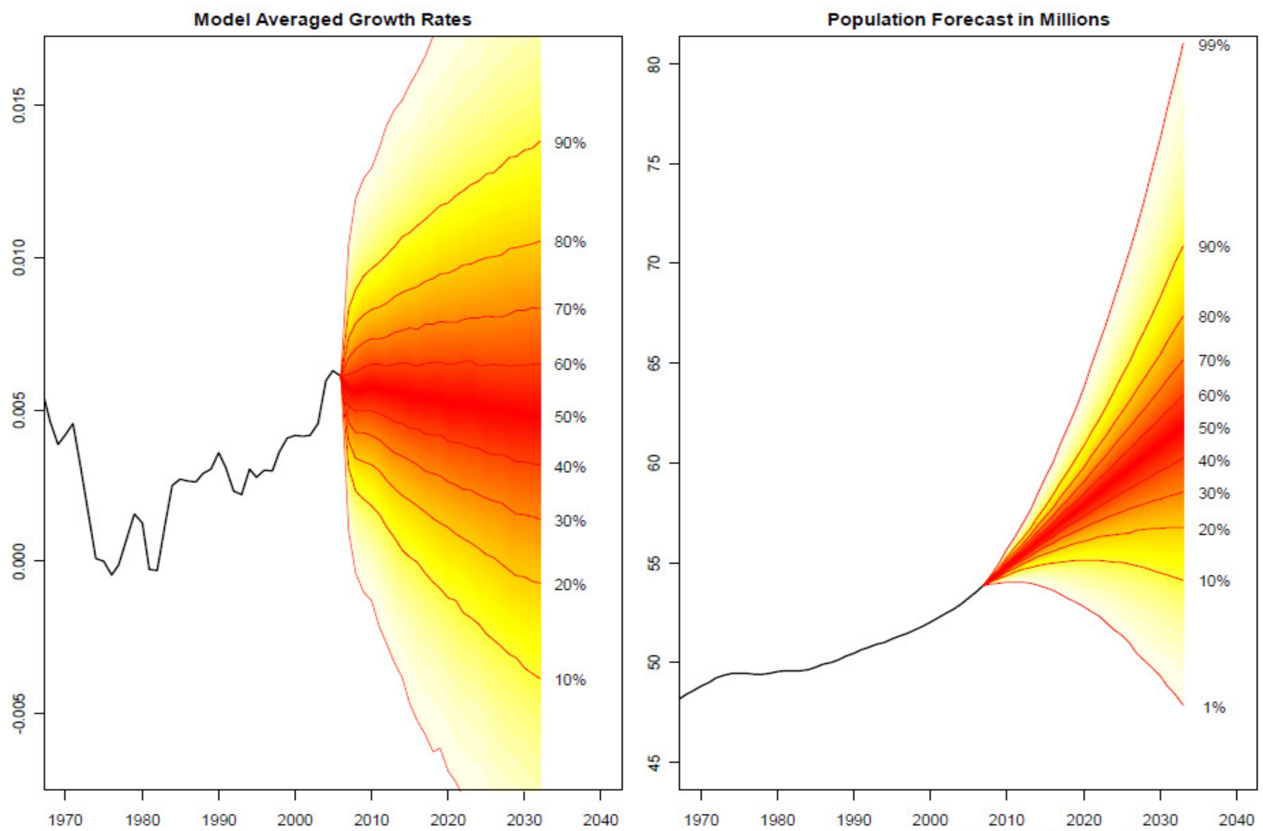
- Population forecast for England & Wales:

Joint Predicted Probability Distribution

AR(p) models



[Abel et al. 2010]

Digression: Expert judgement

- Both estimation and prediction can explicitly take advantage of knowledge of experts in the field
- Elicitation of expert judgements: big challenge
  - – Heterogeneity of expert opinion
    - Another source of uncertainty
    - Can be mitigated by multi-stage framework (Delphi)
  - – Challenges of finding a common language
    - Straightforward, yet precise formulation of questions
    - Avoiding statistical jargon [cf. Bijak & Wiśniowski 2010]

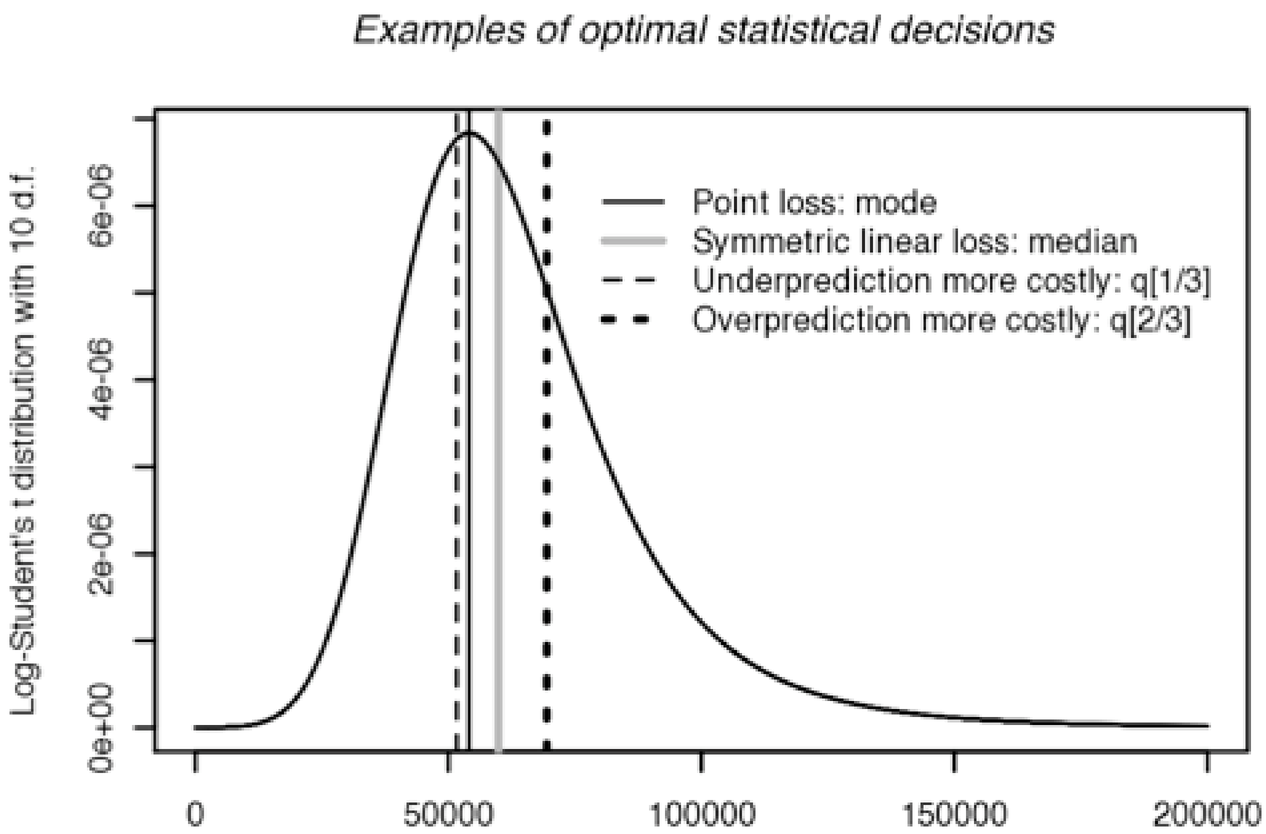
Bayesian decisions

Choice of an optimal decision depends on:

- The underlying probability distribution
  - For estimation: posterior distribution
  - For forecasts: predictive distribution
- The decision setting
  - Loss function $L(w,d)$: cost of making a decision d, when the true state of the world is, or will be, w [DeGroot, 1970]
  - Symmetric versus asymmetric loss functions: In real-life applications, asymmetric are more likely [Lawrence et al., 2006]

Bayesian decisions: example

Stylised migration forecast (log-*t* distribution)



[Bijak 2010]

Bayesian decisions: limitations

Bayesian decisions	Loss function: linear	Loss function: non-linear
Light-tailed distributions	Solutions based on quantiles	Some solutions usually exist
Heavy-tailed distributions		Solutions likely do not exist

[See also: Taleb, 2009]

Common-sense strategies

Conclusions

- Bayesian approach offers a common analytical framework: from estimation through forecasting to decision-making
  - Explicit, coherent description of uncertainty at various levels (parameters, models...)
  - Important role of expert judgement, especially given deficiencies of data (e.g. migration)
  - Potentially more realistic assessment of uncertainty than in traditional (frequentist) forecasting [Bijak 2010]

Future work

- Population estimates and forecasts
  - Coherent methods for including other dimensions (age, sex...) in a multi-regional / multi-state setting
  - *More in two subsequent presentations...*
- Bringing together the qualitative and quantitative analysis within the Bayesian framework
  - Need for a Quantitative-Qualitative dictionary
- Greater involvement of the users of estimates and forecasts: decision analysis
- Complexity and predictability in demography

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All opinions are those of the author only.